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Matching of Catalogues by Probabilistic Pattern Classification

D. J. Rohde[1,2⋆], M. R. Gallagher[2], M. J. Drinkwater[1], K. A. Pimbblet[1]
[1] *Department of Physics, University of Queensland, Queensland, 4072, Australia*
[2] *School of ITEE, University of Queensland, Queensland, 4072, Australia*

ABSTRACT

We consider the statistical problem of catalogue matching from a machine learning perspective with the goal of producing probabilistic outputs, and using all available information. A framework is provided that unifies two existing approaches to producing probabilistic outputs in the literature, one based on combining distribution estimates and the other based on combining probabilistic classifiers. We apply both of these to the problem of matching the HIPASS radio catalogue with large positional uncertainties to the much denser SuperCOSMOS catalogue with much smaller positional uncertainties. We demonstrate the utility of probabilistic outputs by a controllable completeness and efficiency trade-off and by identifying objects that have high probability of being rare. Finally, possible biasing effects in the output of these classifiers are also highlighted and discussed.

Key words: catalogues – astronomical data bases: miscellaneous – methods: statistical

1 INTRODUCTION

The Virtual Observatory (VO) aims to enable new science by enhanced access to data and, more importantly, providing the computing resources required to analyze the data (see www.aus-vo.org for the Australian contribution to the VO). One of the most important capabilities of the VO will be the identification of different observations of the same object. A promising VO tool developed for this task is the Open SkyQuery protocol (Budavári et al. 2004). This tool encourages the combination of many disparate catalogues and will, in the long term, offer a powerful aid to VO enabled science.

A problem presents itself, however, when attempting to combine catalogues with significantly different positional resolutions. A salient example of this is the study of Doyle et al. (2004) who matched the HI Parkes All Sky Survey (HIPASS) (radio) catalogue (Meyer et al. 2004) to the SuperCOSMOS (optical photographic survey) catalogue (Hambly et al. 2001). The HIPASS catalogue is significantly less dense in terms of objects per unit solid angle and has larger positional uncertainties than the SuperCOSMOS catalogue, which by contrast, possesses much more accurate astrometry. This leads directly to objects in the HIPASS catalogue having multiple candidate counterparts to the objects in the SuperCOSMOS catalogue (Fig 1). Previously (Rohde et al. tion) we borrowed the term linkage from the computer science term of record linkage in order to emphasise the statistical aspect of the problem of finding different observations of the same object. In the literature the terms matching, associations and cross-referencing are also used to refer to this (statistical) problem. Here we use the terms matching and linkage interchangeably.

The methods we consider here would be classed as empirical Bayes. Empirical Bayes is a method where frequentist estimators are made of underlying distributions. These estimators are then treated as if they are completely true. We then use theory of probability in order to calculate class membership (match or non-match). The probability that a candidate matches the sparse object is conditioned on (informed by) available information, such as its position, flux and other measurements as well as the observed parameters of other candidates. It is the goal of our work to condition on *all* available information.

Two distinct approaches to catalogue matching or linkage have emerged in the literature, however each fails to use all available information. The first is the generative [1] approach of Sutherland & Saunders (1992) where a number of probability

⋆ E-mail: djr@physics.uq.edu.au

[1] Here the term generative refers to the fact that our model consists of distributions that can 'generate' more data.

density functions associated with each candidate are combined to provide an overall probability. This method is typically applied only in low dimensions of one or two parameters which means that some potentially useful information is ignored. The second approach is a discriminative pattern classification approach that the author has taken in Rohde et al. (2004, tion) and has also been explored in Voisin & Donas (2001). The outputs of these discriminative pattern classification algorithms are binary, that is, they do not provide any indication of the confidence of class membership (match or non-match), however simple extensions such as those outlined in Platt (2000) allow for the output to be converted into a probability. We use the term discriminative to refer to a classifier that gives a probability of class membership. There are arguments that each approach has advantages and disadvantages, we discuss some of these latter.

The discriminative pattern classification approach seems well suited to dealing with high dimensional distributions; however it has a significant drawback in that it calculates the probability that an object matches conditioned on the parameters for that object only. This could lead to inconsistencies such as the sum of all candidate probabilities not summing to one. What is required is a probability that an object is a match conditioned on all available information (i.e. the parameters of all candidate objects and the parameters of the object being matched to). A major result of this paper is the formulation of a method to combine these probabilities in ignorance of candidate information to a probability conditioned on all available information.

From our point of view there are a number of benefits in building a model that produces probabilistic outputs. Firstly our formalism relies on intermediate probabilities to be calculated in order to combine all available information. Secondly the probabilistic output allows difficult matches to be discarded from a scientific analysis. Thirdly we demonstrate how it is possible to use probabilities in order to assist in the search for rare objects. We demonstrate this by finding dark galaxy candidates i.e. HI sources from the HIPASS catalogue with a relatively high probability of having no candidate match. Generative models are inherently probabilistic. Discriminative models can either represent decision (or classification) boundaries or can give probabilities of class membership.

The importance of the distinction between the two techniques is that it is often suggested that it is easier or better to estimate $P(C = 1|x)$, where $C = 1$ refers to class is one (of two) and x is a high dimensional input vector[2] rather than estimating component probabilities $P(x|C = 1)$, $P(x|C = 2)$ and $P(C = 1) = 1 - P(C = 2)$ and applying Bayes' rule. This argument is particularly common when x is high dimensional. We make a comparison of methods applying both and consider the arguments for and against each formulation. We conclude that our final results are largely indifferent to the choice of formulation. Finally we make a case that probabilities are useful in that they allow a trade-off between completeness and efficiency to be achieved.

We further demonstrate the use of probabilities by considering the problem of identifying dark galaxy candidates. In Doyle et al. (2004) a search for HI objects with no optical detection (dark galaxies) was conducted. No strong candidate for objects with this property was found. All HI objects were either accompanied by optical galaxies, or the absence of optical galaxies was satisfactorily explained by the field being obscured by dust or stars. As such the result of Doyle's study was to conclude that HIPASS did not detect any isolated dark galaxies.

If a dark galaxy were to be detected by HIPASS there is high probability that it could not be identified as such because unrelated background optical galaxies fall within HIPASS's large positional uncertainty. The probabilistic output of our classifier is ideal for identifying HI objects with high probability of having no match. We present a list of objects that would be interesting targets for follow up observation.

In Section 2, we introduce and review the problem in detail, drawing a connection between the Sutherland & Saunders (1992) approach and machine learning methods. Section 3 details the algorithms for probabilistic classification. In Section 4 we apply our methods to the HIPASS-SuperCOSMOS problem and evaluate the usefulness of our technique. We discuss biasing limitations on the application of this method to scientific problems in Section 5.

2 THE PROBLEM

Our previous work (Rohde et al. 2004) developed a classifier that predicted if a candidate was a match or a non-match conditioned on the parameters of the candidate in question only. The Sutherland & Saunders (1992) formalism, however, conditions the probability on the parameters of *all* candidates. In this section we outline how it is possible to understand both approaches in the same probabilistic framework. The formulation of the problem presented here is influenced by Fellegi & Sunter (1969) who develop similar ideas for textual data, using the term record linkage.

2.1 Framework for Matching

In simple terms, our problem is resolving which object in a dense catalogue is the counterpart for a given object in a sparse catalogue. Consider a sparse catalogue, A, and a dense catalogue, B. In A, we have a sparse object, a_i, and we would like to know which candidate is the real counterpart for this object in B. We have the candidates $b_{i,1}, b_{i,2}, b_{i,3}, \ldots, b_{i,N_i}$ (Fig 1)

[2] Typically x includes all parameters that might aid in classification in our application. It would include position, magnitude, area, colour, flux and redshift.

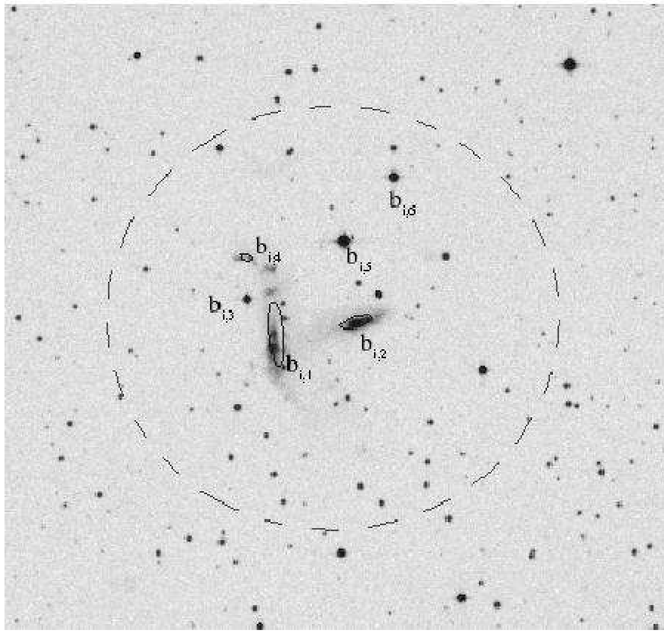


Figure 1. An example of a matching problem. An HI detection from the HIPASS catalogue is being matched to an optical object in SuperCOSMOS. The object from the sparse HIPASS catalogue a_i is located at the centre of this image, the circle represents the 2σ limit of the positional uncertainty. There are a number of candidate optical counterparts from the denser SuperCOSMOS catalogue, $b_{i,1} \ldots b_{i,7}$, (circled).

(each sparse object a_i may have different numbers of candidates denoted by N_i). The catalogue parameters associated with source a_i are represented as a vector α_i, and similarly for the parameters of the source associated with $b_{i,j}$ we use $\beta_{i,j}$.

The cross product $A \times B$ of all possible pairings and consists of the union of two disjoint sets. Formally, $M \subset A \times B$ is the set of all linking pairs of objects and the remaining pairs of objects are non-linking: $U = (A \times B) \setminus M$. We now introduce an indicator variable $z_{i,j}$ which is equal to one if, and only if, $(a_i, b_{i,j}) \in M$ and is zero if $(a_i, b_{i,j}) \in U$.
Our overall aim is to estimate the probability of a match conditioned on all available information.

$$P(z_{i,j} = 1 | \alpha_i, \beta_{i,1}, \beta_{i,2}, \beta_{i,3}, \ldots, \beta_{i,N_i}). \tag{1}$$

Sutherland & Saunders (1992) formulate the overall probability as a normalisation of the likelihood ratio that each individual candidate is a match. The likelihood ratio is given as:

$$L_{i,j} = \frac{P(\alpha_i, \beta_{i,j} | z_{i,j} = 1)}{P(\alpha_i, \beta_{i,j} | z_{i,j} = 0)}. \tag{2}$$

The overall probability can then be calculated :

$$P(z_{i,j} = 1 | \alpha_i, \beta_{i,1}, \beta_{i,2}, \beta_{i,3}, \ldots, \beta_{i,N_i}) = \frac{L_{i,j}}{\sum_{i=1\ldots N_i} L_{i,j} + \kappa}. \tag{3}$$

We include the Sutherland & Saunders (1992) justification for this formula in Appendix A. A priori we expect that each sparse object is likely to match to some dense candidate. If we know the number of candidates then our belief in this is altered. The probability that the dense object has a match when there are N candidates (which we do not know the parameters of) is given by $\frac{1}{N+\kappa}$. The κ parameter allows for a probability to be assigned to the state that there is no match for $\kappa > 0$.

An independence assumption will hold under a wide range of circumstances (that is, the probability of the optical properties of a background object are independent of the properties of the nearest radio source). This allows us to write Equation 2 as :

$$L_{i,j} = \frac{P(\alpha_i, \beta_{i,j} | z_{i,j} = 1)}{P(\alpha_i | z_{i,j} = 0) P(\beta_{i,j} | z_{i,j} = 0)}. \tag{4}$$

A further assumption that position is independent of other parameters may also be reasonable, giving :

$$L_{i,j} = \frac{f(\Delta RA, \Delta Dec)}{k} \frac{P(\alpha'_i, \beta'_{i,j}|z_{i,j} = 1)}{P(\alpha'_i|z_{i,j} = 0)P(\beta'_{i,j}|z_{i,j} = 0)}. \tag{5}$$

Here $f(\Delta RA, \Delta Dec)$ is the probability density function (pdf) on positional uncertainty and k is the density of background objects per unit area, where α'_i refers to the non-positional parameters of α_i and, $\beta'_{i,j}$ refers to the non-positional parameters of $\beta_{i,j}$.

Using the Sutherland & Saunders (1992) approach the problem is solved if we obtain estimators for $P(\alpha_i, \beta_{i,j}|z_{i,j} = 1)$ and $P(\alpha_i|z_{i,j} = 0)P(\beta_{i,j}|z_{i,j} = 0)$. However in Rohde et al. (tion, 2004) a decision rule based on a thresholding of $P(z_{i,j} = 1|\alpha_i, \beta_{i,j})$ is instead used. In the arguments that follow we show how to combine estimators of this discriminative form to produce an estimator conditioned on all available information as in Equation 3. We discuss possible advantages of this approach later.

In this paper we find it useful to convert between probability and a likelihood ratio. The likelihood ratio is the probability of the data given that the object is a match divided by the probability of the data given it is a non-match. In this case the likelihood ratio is equivalent to the posterior odds of class membership. Odds is related to probability by $o = \frac{p}{1-p} = \frac{1}{p^{-1}-1}$, where p is probability and o is the odds. The likelihood ratio $L_{i,j}$ is equivalent to the odds of $b_{i,j}$ being a link in the absence of candidate information, divided by the prior odds $(\frac{P(z_{i,j}=1)}{P(z_{i,j}=0)})$. The Sutherland & Saunders (1992) result presented here in Equation 2 and 3 is an expression of the *probability* that a_i links to $b_{i,j}$ given all information. The probability is calculated as the normalisation of the *odds* of each candidate in ignorance about candidate information. The conversion to odds given a probability is given by $L_{i,j} = \frac{P(\alpha_i,\beta_{i,j}|z_{i,j}=1)}{P(\alpha_i,\beta_{i,j}|z_{i,j}=0)} = \frac{\frac{P(z_{i,j}=0)}{P(z_{i,j}=1)}}{P(z_{i,j}=1|\alpha_i,\beta_{i,j})^{-1}-1}$ (This can be verified by substituting Bayes' rule for $P(z_{i,j} = 1|\alpha_i, \beta_{i,j})$). Applying the Sutherland & Saunders (1992) result we find:

$$P(z_{i,j} = 1|\alpha_i, \beta_{i,1}, \beta_{i,2}, \cdots, \beta_{i,N}) = \frac{\frac{\frac{P(z_{i,j}=0)}{P(z_{i,j}=1)}}{P(z_{i,j}=1|\alpha_i,\beta_{i,j})^{-1}-1}}{\sum_{k=1..N} \frac{\frac{P(z_{i,j}=0)}{P(z_{i,j}=1)}}{P(z_{i,k}=1|\alpha_i,\beta_{i,k})^{-1}-1} + \kappa} \tag{6}$$

which simplifies to

$$P(z_{i,j} = 1|\alpha_i, \beta_{i,1}, \beta_{i,2}, \cdots, \beta_{i,N}) = \frac{\frac{1}{P(z_{i,j}=1|\alpha_i,\beta_{i,j})^{-1}-1}}{\sum_{k=1..N} \frac{1}{P(z_{i,k}=1|\alpha_i,\beta_{i,k})^{-1}-1} + \frac{P(z_{i,j}=1)}{P(z_{i,j}=0)}\kappa.} \tag{7}$$

Using a discriminative probabilistic classifier, it is possible to estimate $P(z_{i,j} = 1|\alpha_i, \beta_{i,j})$, and using Equation 7, to combine all candidate information to arrive at a probability of a match conditioned on all available information. We use this rule to calculate probabilities using the discriminative method below.

While we would normally estimate $P(z_{i,j} = 1|\alpha_i, \beta_{i,j})$ directly it is useful to consider the component probabilities to this estimator.

$$P(z_{i,j} = 1|\alpha_i, \beta_{i,j}) = \frac{P(\alpha_i, \beta_{i,j}|z_{i,j} = 1)P(z_{i,j} = 1)}{P(\alpha_i, \beta_{i,j}|z_{i,j} = 1)P(z_{i,j} = 1) + P(\alpha_i, \beta_{i,j}|z_{i,j} = 0)P(z_{i,j} = 0)}. \tag{8}$$

It is a well justified assumption for our problem that $P(\alpha_i, \beta_{i,j}|z_{i,j} = 0) = P(\alpha_i|z_{i,j} = 0)P(\beta_{i,j}|z_{i,j} = 0)$. However this is not taken into account when $P(z_{i,j} = 1|\alpha_i, \beta_{i,j})$ is estimated using a standard algorithm. This assumption is not utilised Rohde et al. (tion, 2004).

2.2 The Two Approaches to Classification

Classification problems can be approached in two different ways. The first is to estimate the probability $P(C = 1|x)$[3]. We describe this as the discriminative approach. Sometimes the word discriminative is used to refer to a decision boundary however by simple decision theoretic arguments, this is simply a threshold of $P(C = 1|x)$ (Duda et al. 2000). Here we use the word discriminative more generally to refer to $P(C = 1|x)$. Discriminative classification is the basis of many methods including neural networks, Platt calibrated (Platt 2000) Support Vector Machines (SVMs) (Vapnik 1995) and logistic regression (Hosmer 2000).

The alternative is to calculate the component probabilities $P(x|C = 1)$ and $P(x|C = 2)$ and $P(C = 1) = 1 - P(C = 2)$ and then apply Bayes' rule in order to obtain $P(C = 1|x)$. We will describe such techniques as generative methods. This approach is used in, for example, nearest neighbours' approaches to classification.

[3] $P(C = 1|x)$ is the probability that x belongs to class 1 conditioned on x.

In machine learning it is more common to estimate $P(C = 1|x)$ rather than all the component probabilities. Vapnik (1995) argues that using the (simple) discriminative formulation is a fundamental principle of statistics. Hand (1996) remarks that discriminative models such as neural networks are successful because they find an intermediate position between simple parametric models and complex generative density estimation models such as nearest neighbours [4].

The above framework for matching allows the probability of a match to be calculated from either generative or discriminative estimators. Competing arguments for the merits of each approach exist:

(i) The generative model $P(x|C = 1)$, $P(x|C = 2)$ and $P(C = 1) = 1 - P(C = 2)$ allows the likelihood ratio method to be used directly Equations (2-5). The simplifying independence assumption can be introduced into Equation 4.

(ii) The discriminative model $P(C = 1|x)$ is in agreement with the *principle* suggested in Vapnik (1995) and takes the intermediate complexity which is argued as positive in Hand (1996) (between parametric and nearest neighbour methods), and is generally the more common machine learning approach. However the independence assumption in Equation 8 must be ignored and the model is left underconstrained.

In this paper we apply both a generative and a discriminative model. The generative density estimation is performed by using a high dimensional Gaussian Mixture Model fitted to the data using the Expectation Maximisation (EM) algorithm. This produces estimates of $P(\alpha_i, \beta_{i,j}|z_{i,j} = 1)$ and $P(\alpha_i|z_{i,j} = 0)P(\beta_{i,j}|z_{i,j} = 0)$. The Sutherland & Saunders (1992) formalism can be applied by using Equation 3 and Equation 4.

The discriminative model is fitted using an SVM with Platt Calibration Platt (2000). This produces an estimate of $P(z_{i,j} = 1|\alpha_i, \beta_{i,j})$ which can be combined with the probabilities of all the other candidates $(b_{i,1}, \cdots, b_{i,N_i})$ using Equation 7.

3 ALGORITHMS

A number of different non-parametric approaches are available for the problems of density estimation and estimating class probabilities. Both approaches are dominated by the use of the principle of maximum likelihood.

3.1 Density Estimation

According to the formulation in Section 2, our problem is solved if we obtain a good estimator of the densities $P(\alpha_i, \beta_{i,j}|z_{i,j} = 1)$ and $P(\alpha_i|z_{i,j} = 0)P(\beta_{i,j}|z_{i,j} = 0)$.

One method for density estimation is the use of the k-nearest neighbours averaging. This method has a very large number of effective parameters that increase in proportion to the size of the dataset. The k-nearest neighbours averaging is very discontinuous which seems undesirable. The kernel density estimation technique overcomes this by smoothing the output using a Gaussian kernel. The kernel method also has a very large number of effective parameters which means there is a high chance of producing an overly complex model that fits the idiosyncrasies of the data, this is of particular concern when applying the method to high dimensional problems. Discussion and comparison of these techniques can be found in Hastie et al. (2001).

The most appropriate method for this is to fit a semi-parametric model such as a Gaussian mixture model to the data using maximum likelihood. This can be achieved using the Expectation Maximisation (EM) algorithm. The EM algorithm allows the analytical form for maximum likelihood estimators for mean and standard deviation to be used on a mixture model by introducing the concept that every mixture component has a certain responsibility for every data point. The E step calculates the responsibilities where the M step maximises the likelihood (McLachlan & Krishnan 1997). EM is an iterative hill climbing algorithm on the likelihood function. In this paper we use the EM implementation provided by the Netlab software package (Nabney 2003). It is possible for the EM algorithm to converge to a poor local optimum of the likelihood function. For this reason it is often necessary to attempt the optimisation with different initial conditions. We follow the recommendation in Nabney (2003) to use the k-means clustering algorithm in order to set the initial parameters of the EM algorithm.

3.2 Class Probabilities

Neural networks are multi-parameter models that can be fit to labelled data using the principle of maximum likelihood to estimate class probabilities. When a neural network is fit to targets that are binomial class probabilities, the use of a 'cross-entropy' error function leads to a maximum likelihood estimate of the class probability (Bishop 1995). In previous work we found that neural networks showed sub-optimal performance for achieving high classification rates on the SuperCOSMOS-HIPASS matching problem (Rohde et al. tion).

Support Vector Machines (SVMs) are primarily classification algorithms. The SVM projects the input vectors into a high dimensional feature space and finds a separating hyperplane. The separating hyperplane is chosen using the criterion of maximal margin which means that the distance of the plane to the closest points (of opposing classes) is maximised (Cristianini & Shawe-Taylor 2000) (Vapnik 1995). In this study we use the SVM lite software (Joachims 1998) in order to

[4] Nearest neighbours is a conceptually simple algorithm however it has a large number of effective parameters (proportional to the number of datapoints) this can lead to complex decision boundaries.

fit the model. The SVM output is a real number (g), with the sign representing the side of the plane and the magnitude represents the distance from the plane. A binary classifier is produced by placing a threshold of zero on g. A probabilistic classifier can be obtained by considering $P(C=1|g)$. We follow the method proposed in Platt (2000) where a logistic sigmoid is fitted to $P(C=1|g)$ using the principle of maximum likelihood in order to obtain probabilistic outputs -

$$P(C=1|g) = \frac{1}{1+e^{w_1 g + w_2}}. \tag{9}$$

The parameters w_1 and w_2 are adjusted as part of fitting the model, in order to maximise the likelihood of the sigmoid. We again use the Netlab package to do this using a quasi-Newton optimisation algorithm.

The details of this calibration process are as follows. Platt calibration involves a three-fold-cross-validation procedure for fitting the sigmoid. The SVM is fitted to two thirds of the training data, the remaining third is used to determine w_1 and w_2. This procedure is repeated three times and the average value of w_1 and w_2 is then used. In order to avoid a very rapid transition of the output probability from zero to one, Platt recommends training on non-binary targets. Hence, rather than have training points labelled as 1 for $C=1$ and 0 for $C=2$, they have non-binary values. Instead we use $\frac{M_1+1}{M_1+2}$ for $C=1$ (M_1 is the number of training examples drawn from class 1) and $\frac{1}{M_2+2}$ for $C=2$ (M_2 is the number of examples drawn from class 2). This is justified using regularisation arguments in Platt (2000).

4 APPLICATION OF METHODS TO SUPERCOSMOS & HIPASS

The matching of the HIPASS radio catalogue (Meyer et al. 2004) to the optical SuperCOSMOS catalogue (Hambly et al. 2001) is a difficult problem due to the poor positional uncertainty on the HIPASS catalogue ($\sigma \approx 1$ arcmin). The use of external redshifts from the NASA Extragalactic Database and the Six Degree Field Survey (Wakamatsu et al. 2003) along with human judgment are able to match approximately half of this catalogue known as HOPCAT (HIPASS Optical Catalogue) (Doyle et al. 2004). In previous work a (non-probabilistic) binary classifier was applied to this training data (Rohde et al. 2004). A cross-validation process gave very good overall performance 99.12 per cent. However this involved only applying the classifier to a single case at a time, we however made the additional assumption that exactly one candidate could be a match. The binary SVM however did not incorporate this assumption. On unmatched date the binary SVM found 1209 new matches. In 1012 other cases the classifier gave an ambiguous result either selecting zero or multiple matches. In this section we demonstrate the ability of probabilistic approaches to enforce a constraint that every sparse object has exactly (or at most) one match.

4.1 Model Validation

We have available two broad approaches for calculating probabilities. How can we then discriminate between which is the better of the two approaches? What makes a probability a good probability? This turns out to be a very difficult philosophical question. The definition of probability is the source of one of the most celebrated disputes in science (Howie 2002) and results in two competing paradigms for statistics, frequentist and Bayesian.

Rather than delving deeper into this philosophical issue we offer some heuristic tests to evaluate the quality of probability. The fact that a number of intuitive tests exist results from the fact that there is no clear cut method for evaluating the quality of the probabilities produced. We present the measures for the three methods considered, but the nature of the problem only allows us to make intuitive statements about which method is better.

It is standard practice to put part of the data aside in a test set in order to evaluate properties of the model. Our model validation involves training on 75 per cent of the data, 9941 training vectors (i.e. 1356 positive examples and 8585 negative examples) and testing on the remaining 25 per cent 3304 vectors (453 positive and 2851 negative examples).

The measures of probability quality that we consider are:

(i) Classification rate or percentage correctly classified. One of the main reasons for producing a probability is to correctly match our datasets, testing the classifiers ability to correctly classify a test set has obvious intuitive appeal. This test however gives equivalent results for any monotonic increasing function of the probability. We make this test by counting how often the classifier assigns the highest probability to the correct match on our test set.

(ii) Calibration is the property that if all of the examples where $P(C=1|x) \approx k$ are binned then the proportion of objects in that bin belonging to class one should be approximately k. For example if $P(C=1|x) = 0.2$ then the classifier is well calibrated when 20 per cent of objects with this value of x belong to class 1 . Calibration is only part of the picture as perfect calibration can arise if the classifier makes dishonest predictions in order to obtain frequencies in agreement with predicted probabilities (DeGroot & Fienberg 1982). The property of calibration is also independent of classification rate.

(iii) A number of scoring rules have been introduced in order to rank probabilities. The most widely used is the Brier (1950) score which is a mean squared error statistic. For example if the classifier predicts a value of 0.9 and it is a match, then the contribution to the Brier score is $0.1^2 = 0.01$. If for example it is not a match, then the contribution is $0.9^2 = 0.81$.

Note that minimising (i) and (iii) alone can result in overfit models that are overly complex and generalise poorly. Cali-

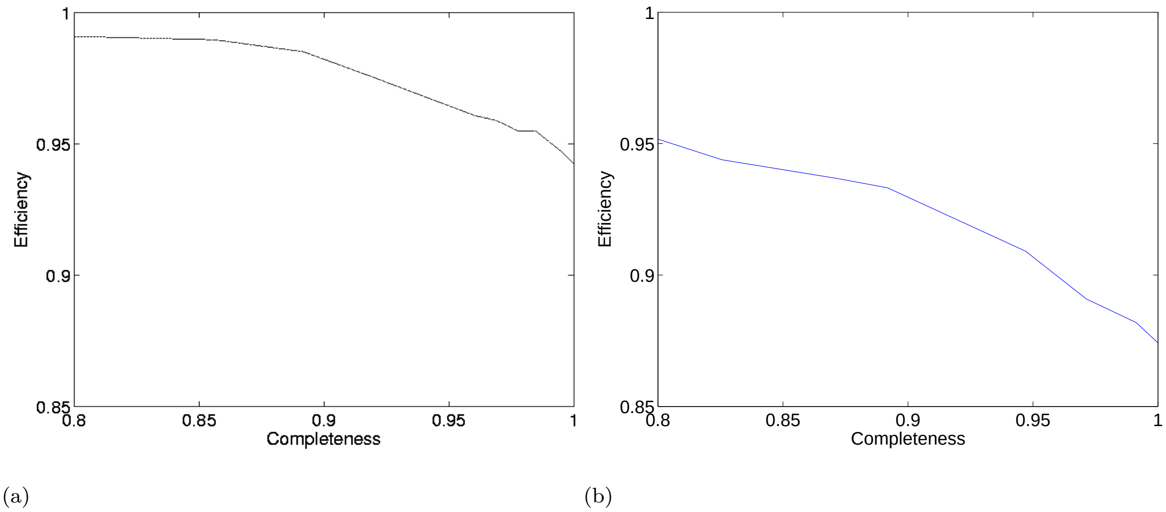


Figure 2. Trade-off between completeness and efficiency of (a) the discriminative SVM based classifier and (b) the generative mixture model based classifier. The SVM based classification is consistently better than the mixture based classification. In practice a decision about the trade-off between completeness and efficiency must be made. While completeness can be sacrificed for efficiency dropping below a completeness of 0.8 brings very marginal benefits. At a completeness of 1 reasonably high efficiencies are obtained.

bration becomes important when there is an inevitable proportion that is misclassified. In general minimising the classification rate will also cause the Brier score to be lowered.

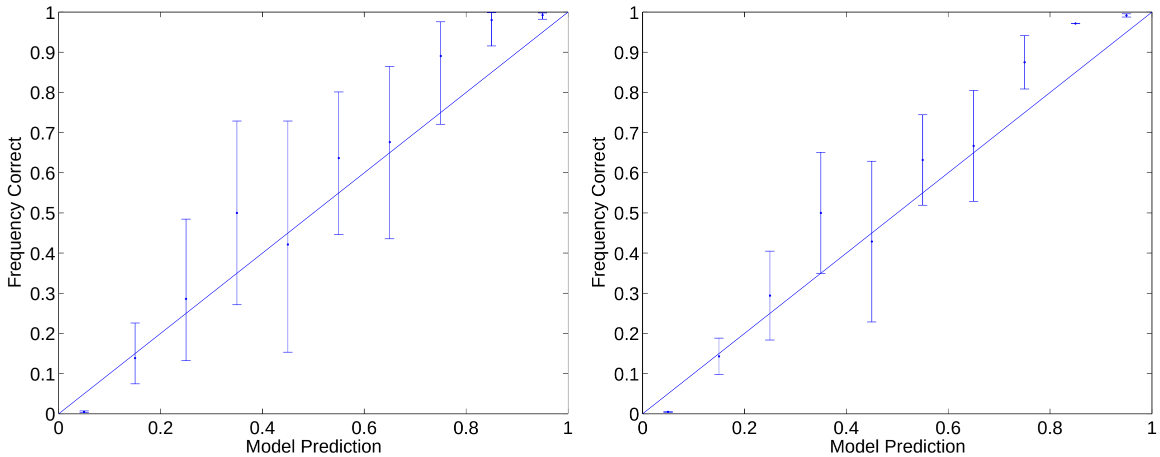
One of the advantages of using probabilistic outputs is that it allows for cases where it is difficult to determine a match to be discarded. In pattern classification the Receiver Operator Characteristic (ROC) curve is commonly used in order to describe the trade-off between incorrect classifications, false positives and false negatives (Duda et al. 2000). False positives are negative examples that have been incorrectly labelled positive, and similarly false negatives are positives that have been incorrectly labelled negative. This can be directly interpreted as a trade-off between completeness and efficiency where completeness is the proportion of sparse objects one finds matches for, and efficiency is the number of correct matches in the dataset. The optimal trade-off will be application dependent. We see the trade-off for our dataset for our two models in Fig 2. A perfect curve would be one where the efficiency was 1 for every completeness between 0 and 1. From these plots it is apparent that the SVM achieves better classification than the mixture model for all possible completenesses.

We evaluate calibration by the use of reliability diagrams (Caruana & Niculescu-Mizil 2005). Reliability diagrams are produced by binning the output of the classifier and testing if the class frequencies (on the test set) are in agreement with the model's prediction. The SVM and mixture reliability diagram is shown in Fig 3. The error bars are 90 per cent credible regions. Credible regions are the Bayesian alternative to confidence intervals. In our example a clear advantage is that the credible region takes into account a priori knowledge that probabilities can only be between 0 and 1. A persuasive argument for credible regions over confidence intervals is found in Jaynes (1983). Confidence regions are calculated using a uniform Beta conjugate prior. See Bernardo & Smith (1994) for details. The error bar indicates a region with 90 per cent probability of containing the frequency with which probabilities are assigned to a particular class. The centre point is the posterior median. Both the SVM and the mixture model exhibit satisfactory calibration. This is demonstrated by the fact that the probabilities in general fall along the diagonal line. It is noteworthy that as the SVM classifies more objects correctly, there is less data available to calibrate the probabilities. This causes the error bars to be much larger for the central probabilities in the SVM reliability diagram compared to the probability values near zero or one, this is also true but less pronounced for the mixture model.

Finally we consider the Brier score which in some way gives a combined indication about calibration and classification performance. The SVM has a Brier Score of 0.0168; the mixture model has a Brier score of 0.0863. Presumably the SVM's better classification is primarily responsible for this, at a completeness of 100 per cent the SVM correctly classified 94 per cent where the mixture model classified 87.5 per cent correctly. The benefit of good calibration is only apparent if errors are unavoidable.

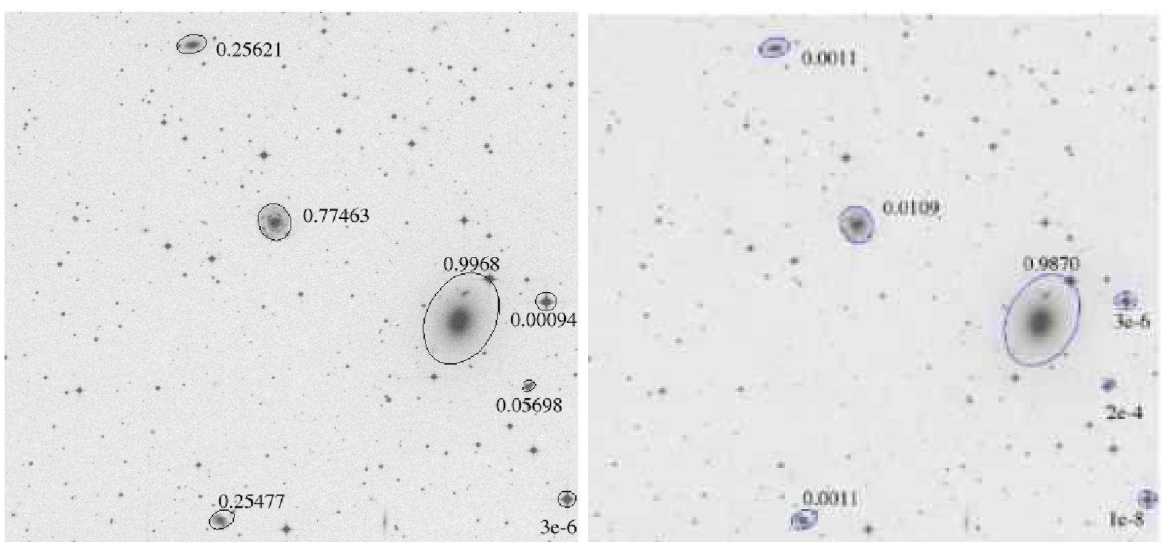
4.2 Examples of Classification

While the empirical validation of our model is helpful, it is also useful to include examples so that we can see if the predictions agree with intuition. In order to do this we use the discriminative SVM to calculate probabilities for a number of galaxies in order to make a qualitative assessment. It has already been shown that the Support Vector Machine can separate the two classes with high accuracy(Rohde et al. tion) (for those cases that were not discarded), so we are not interested in determining the correct match per se. Rather we are interested in illustrating how the combination of candidate information changes our probabilities. We can see this in Fig 4 where in (a) the probabilities are produced in ignorance of candidate information these



(a) (b)

Figure 3. Reliability diagrams for (a) the discriminative SVM classifier and (b) the generative mixture model classifier. Good performance is indicated by the points lying on or near the diagonal line.



(a) (b)

Figure 4. The probabilities in the absence of candidate information (a) and in the presence of candidate information (b). Note that candidate information makes the probabilities well behaved in the sense that they now should sum to one (or less that one if the identification rate is less than 1).

probabilities are not well behaved in that they do not sum to one. In Fig 4 (b) candidate information has been incorporated. The galaxy that was initially thought to be a 77 per cent chance of a match is revised to a mere 1.1 per cent when all candidate information was incorporated.

Another point that we would like to show qualitatively is that the class predictions are appropriately *confident* on easy examples and *cautious* on hard cases. In Fig 5 we show a number of successful classifications, sometimes in difficult circumstances. Appropriately the classifier quantifies difficult cases using low probabilities. It is particularly impressive that the classifier distinguishes Fig 5 (a) and (b) as these correspond to two different HI sources in very close proximity. It is pleasing to see the probabilities are relatively low (less than 70 per cent). In Fig 5 (c) and (d) it is encouraging that the classifier assigns some probability to the background galaxies, but is able to place the bulk of the probability on the correct label.

In Fig 6 we examine cases where the classifier has assigned the highest probability to an incorrect match. In Fig 6 (a) and (b), while incorrect, the classifier's prediction seems reasonable - the probabilities are low indicating a high degree of uncertainty. Qualitatively, there appears good reason to be unsure, we should expect that some of the time the match is not

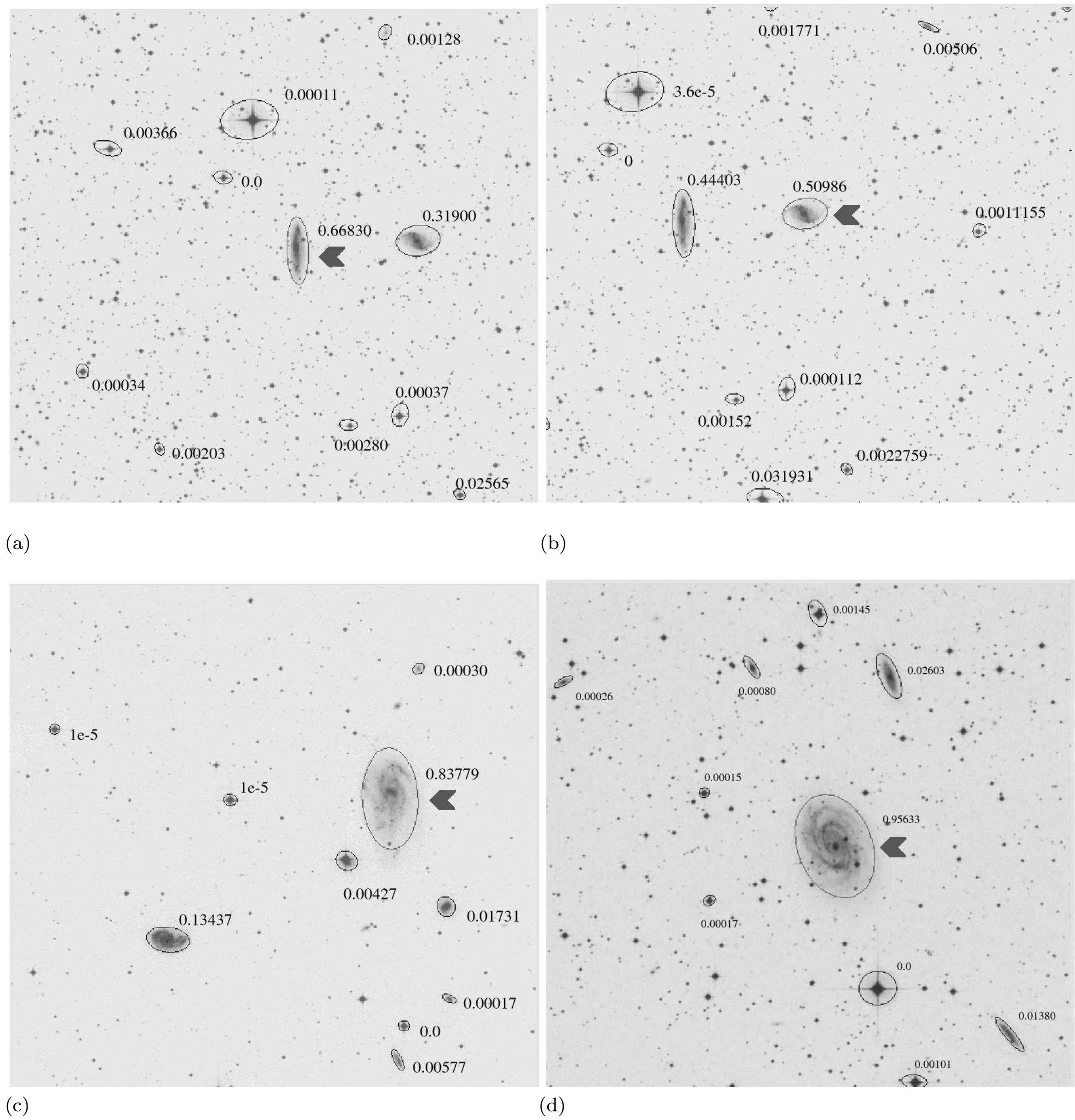


Figure 5. Some examples of correct matches (match marked with an arrow). In each case optical redshifts are able to select the matching optical galaxy and eliminate the non-linking optical galaxies. Two HIPASS detections are in close proximity in (a) and (b) and the classifier is able to classify correctly, but more importantly offer considerable qualification to the classification. In (c) again we obtain a correct match with an understandable level of qualification because of the other significant candidate. In (d) the classifier chooses the correct link confidently deciding that the number of background galaxies are relatively poor candidates.

the highest probability. In Fig 6 (c) the classifier is confused by the presence of two bright objects that we know from optical spectroscopy are unrelated to the HI source. This problem is even worse in Fig 6 (d) where the classifier is apparently certain about an incorrect match. Effectively the classifier has told us that an event we know occurred, cannot possibly occur! The reason for this, is that we had a very extreme case present in the test data for which there is no similar case in the training data, the background galaxy in (d) is a very large and significant galaxy. A better situation would be for the classifier to assign some probability to this event. In near separable data, such as this, calibration is a difficult problem as little data lies in the unsure region where probabilities are between 0 and 1.

4.3 Dark Galaxy Candidate Search

One of many goals of the HIPASS survey is to identify low surface brightness (LSB) and dark galaxies, that is HI sources without any (visible) optical counterpart. This was carried out using the subset of radio galaxies matched by Doyle et al. (2004)

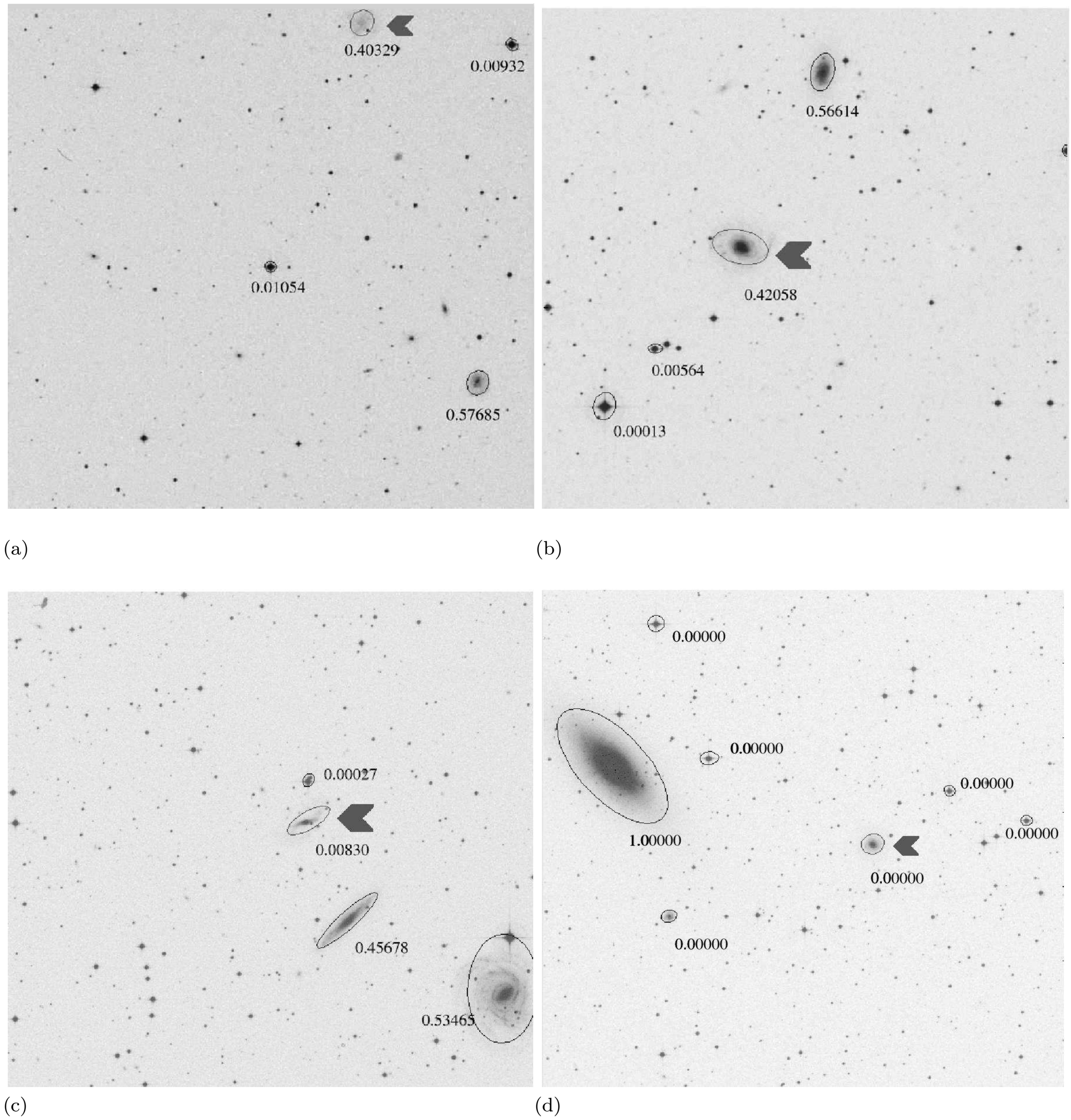
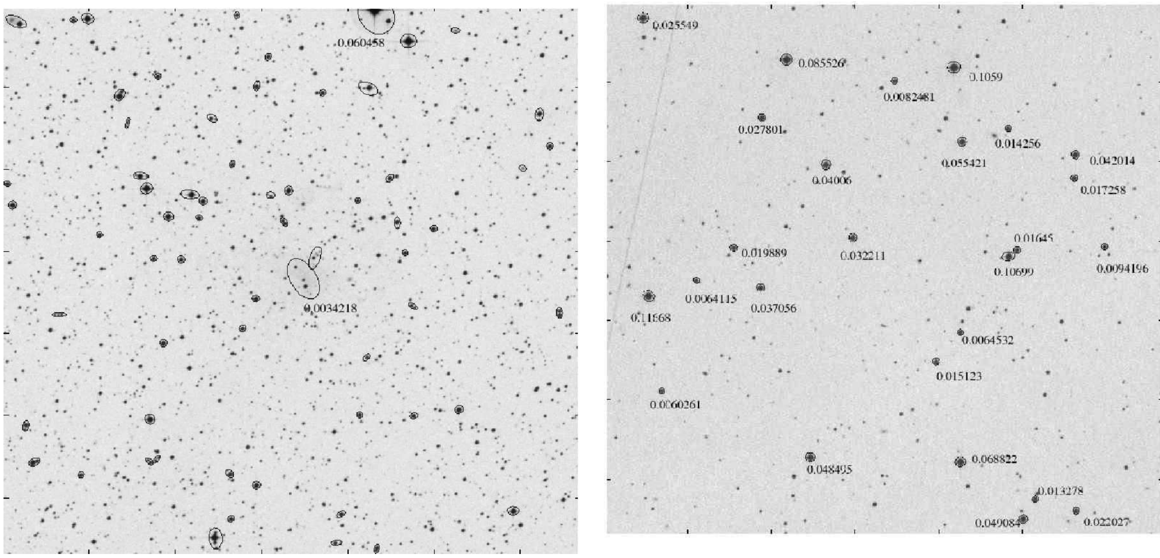


Figure 6. Some examples of incorrect matches (match marked with an arrow). Two qualitative issues are relevant here: are these difficult cases? Does the classifier predict a reasonable probability to the correct match, and only slightly greater for the incorrect match? In (a) and (b) we get an encouraging result where the result is qualified by the fact that the highest probability is quite low, and the true link is only slightly less likely. In (c) and (d) performance is hampered by the presence of very large and bright background objects. In (c) the highest probability is quite low, but disappointingly we obtain a low probability for the true match. In (d) within the precision of our calculations we are 'certain' about an incorrect link.

to SuperCOSMOS known as the HOPCAT catalogue. The HOPCAT catalogue contained a number of HIPASS detections where there are no optical counterparts in the field, however it was found that the absence of any optical sources could be satisfactorily explained by dust or stars obscuring the view or by a false HIPASS detection. In all other cases there was at least one possible match, so it was concluded that there was no evidence in HOPCAT for the detection of dark galaxies (Doyle et al. 2004).

The failure of HOPCAT to detect isolated dark galaxies might be attributed to the large positional uncertainty of HIPASS together with the relative depth of SuperCOSMOS, leading to a very low probability of identifying *isolated* dark galaxies. Dark galaxies that happen to lie near a background optical object cannot be identified using HIPASS, however follow up observation using high resolution radio HI may be able to identify these objects. This telescope time is precious so it is a worthwhile application to use the probability of no optical counterpart as predicted by our classifier as a method for selecting targets.

We primarily use the same criteria as HOPCAT for the consideration of HIPASS sources. There must be an extinction



(a) (b)

Figure 7. Two examples of the probabilities of candidates around dark galaxy candidates. In (a) J105732-48 which is classed as a good guess in HOPCAT - viewing the image in ds9 and varying contrast makes it seem credible that the central object is a match. It is interesting nonetheless as the galaxy is quite diffuse. Probabilities that are less than 0.0005 are not shown. In (b) J0532-78 no strong candidate counterpart exists. Relatively high probability is assigned to the stars, partially reflecting the inability of the classifier to identify stars with absolute confidence. However it is an affect of the absence of any reasonable candidate causing substantial probability mass to be assigned to the stars. Intuitively, when we further condition on our own belief that these stars cannot be matches to the HI source the probability that this object is rare increases to be greater than the 0.01 as previously calculated.

in the B_j passband < 1 mag and the object must not be on the galactic plane or obscured by stars. However we relax the assumption that there must be no galaxies in the field to simply include fields where the probability of no match is high. This procedure selects fields where there are few candidates and these objects are more consistent with being background objects than HIPASS counterparts.

In order to detect dark galaxy candidates we must set the identification rate κ to greater than 0; we rather arbitrarily set $\kappa = 1.6 \times 10^{-4}$ or $\kappa \frac{P(z_{i,j}=1)}{P(z_{i,j}=0)} = 0.001$. The choice of κ is very subjective, it depends on a priori belief in the existence of dark galaxies. However regardless of the choice of κ the output probability is a monotonic function of the probability of a dark galaxy, so this is no obstacle to creating a list of best candidates for follow up observation.

When $\kappa > 0$ the probabilities for each candidate in a field will not sum to one. The probability of no match (dark galaxy) is one less the sum of all candidate probabilities. The HI sources that are most likely to be dark galaxies will then be those that have few good candidate matches. Using this procedure we produced a list of dark galaxy candidates and the probability that they have no match in Table 1.

The list contains the 30 objects with highest probability of being a dark galaxy according to our classifier. They also satisfy the criteria of having the extinction in the B_j passband < 1 mag and have not previously been eliminated as candidates by Doyle et al. (2004). The description column contains comments obtained from doing a visual inspection of each field. Fields containing small galaxies would be particularly interesting to follow up with high resolution HI observations. Follow up observations could help to determine (i) if there are more false HIPASS detections, (ii) locate dark galaxies or (iii) identify HIPASS sources that are very small or faint in SuperCOSMOS. It would also have implications for the reliability of the HOPCAT catalogue.

Two fields containing examples of dark galaxy candidates are shown in Fig 7. The images are most strongly characterised by the lack of large and significant galaxies. The description column of Table 1 indicates this is typical; the fields containing larger galaxies may have optical properties that indicate they are unlikely to contain significant amounts of HI or perhaps, more likely, are cases where the classifier performs badly.

5 BIASING LIMITATION

Ideally the matched output would recover the underlying distribution $P(\alpha, \beta | z = 1)$ in the output it produces, however we note here that biasing effects are possible.

A development in this paper is the use of probabilities conditioned on all available data. It is tempting to pair every sparse object with the most likely dense candidate and then to act as if this combined catalogue is completely true in the analysis to follow. This has the advantage that we find the most likely match for *every* sparse object. In contrast the binary

Name	Probability	Description	Class	Sint(Jy km s^{-1})	Velocity (km s^{-1})
J1057-48	0.94	LSB candidate match in centre	gg	101.7	597.6
J2350-40	0.58	Near empty field	no-vel	33.5	1698.4
J2355-39	0.47	Several faint galaxy	no-vel	21	263.4
J0033-09	0.27	Near empty field	no-vel	4.6	2751.5
J2251-20	0.15	Two good candidate matches	no-vel	14.8	3166.7
J1341-02	0.12	Near Empty Field	no-vel	3	8820.9
J2250+00	0.11	LSB galaxy in centre	no-vel	2.8	1696.5
J2351-40	0.11	Several galaxies near centre	no-vel	7.6	344.3
J1225-06	0.10	Some galaxies near centre	no-vel	6.2	1231.4
J0623-42	0.09	Galaxy near centre; crowded with stars!	no-vel	10.7	2259.3
J1435-17	0.07	LSB at high separation	no-vel	9.1	1576
J1227+01	0.05	LSB near centre	no-vel	9.1	1576
J1024-12	0.05	LSB near centre	no-vel	31.9	1292.4
J0951+01	0.04	LSB near centre	no-vel	2.6	623.3
J0249+01	0.03	LSB near centre	no-vel	2.4	2936
J1045-83	0.03	Spiral near centre; crowded field	no-vel	9.73	2123.2
J0214-13	0.02	Good candidate match near centre	no-vel	3.2	5812.3
J1438-18	0.02	Good guess near edge	no-vel	12.3	2562.7
J0013-26	0.02	Near empty field	no-vel	3.7	4871.6
J2207-75	0.02	LSB candidate near centre	no-vel	2.9	2796.9
J2150-23	0.02	Faint galaxies	no-vel	5.2	2346.5
J1334-12	0.02	Faint galaxies	no-vel	2.2	1503.7
J0958-85	0.02	Faint galaxies; crowded with stars	no-vel	2.1	1976.1
J2331+01	0.02	Two reasonable candidate matches	no-vel	7	1271.5
J1347-30	0.01	Several reasonable candidate matches	no-vel	53.1	4358.3
J1812-74	0.01	Faint galaxies; crowded with stars	no-vel	3.8	31991.7
J0946-74	0.01	Good candidate match; crowded with stars	no-vel	46.9	1152.9
J0648-84	0.01	LSB good candidate match	no-vel	3.5	5287.9
J0909-83	0.01	LSB good candidate match	no-vel	3.9	2033.1
J0532-78	0.01	Some faint galaxies; crowded field	no-vel	4.1	6103

Table 1. Dark Galaxy Candidates. The probability column refers to the predicted probability that this HI source has no counterparts and the description column is from visual inspection. As explained in the text the interpretation of the probability is dependent on a subjectively determined κ which is higher for higher beliefs in dark galaxies. The abbreviation LSB is used for low surface brightness galaxies. HOPCAT Class describes how class was determined in HOPCAT; gg means the object was a good guess i.e. it was judged that there was only one likely candidate; no-vel means that many galaxies are present in the image, but redshift information is insufficient to determine a match. The final columns are the integrated flux measured from HIPASS, and the Velocity from HIPASS. The average flux of HIPASS is 15.76 Jy km s^{-1} and the mean velocity is 3275.14 km s^{-1} over the entire sample.

SVM produced 1012 ambiguous results that had to be discarded. However we note here that keeping all information comes at a price: choosing the most likely candidate makes the output distributions distorted. We illustrate this effect with a simplified example.

Consider the simplest possible problem where we match objects using a single parameter ΔRA. We only consider objects with a ΔRA less than 5 arcsec. The distribution for ΔRA for matched objects has a Gaussian distribution of mean 0 arcsec and standard deviation of 1 arcsec and the non-matching objects are uniform between -5 and 5 arcsec. For each field there is exactly 1 match and exactly 1 non-match. The distribution of matches and non-matches is represented in Fig 8; the overlap shown is the Bayes risk i.e. the unavoidable error rate when classifying.

The analogue of the binary classification approach in this situation is to accept a match if, and only if, there is exactly one object in the accept region (for the purposes of this discussion we will accept matches between -3 and 3 arcsec). If there are multiple or zero objects in this region then the data is discarded. The analogue of the probabilistic method discussed here is to take the most probable object as being the match (and act as if this is completely true); no data is ever discarded.

We generate 10^6 random classification problems. This is done by drawing one sample from the Gaussian and one from the uniform distribution. The two different approaches are applied, the most likely match always assigns a class, the binary classifier will either assign a class or discard the data. We then consider the output distribution for the false positive distribution and the false negative distribution. Fig 9 (a) shows the recovered output using the thresholding decision rule, the output is near Gaussian but has the tails cut off at < -3 and > 3 arcsec. Also 30 per cent of data was discarded. Of the data that was retained only 0.4 per cent was erroneous: the false negatives (Fig 9 (d)) are the lost tails and, the false positives represent a uniform distribution which has been added to our recovered output. This is a relatively simple situation with a near Gaussian distribution recovered at the cost of a lot of data being discarded.

When selecting the most probable match no data is discarded, however the error rate increases to 16 per cent. Moreover the distribution of errors appears more complex. The output obtained is not Gaussian (Fig 9 (b)). The false positives are clustered around low proximity (Fig 9 (e)); and the false negatives (Fig 9 (h)) show a bi-modal distribution representing

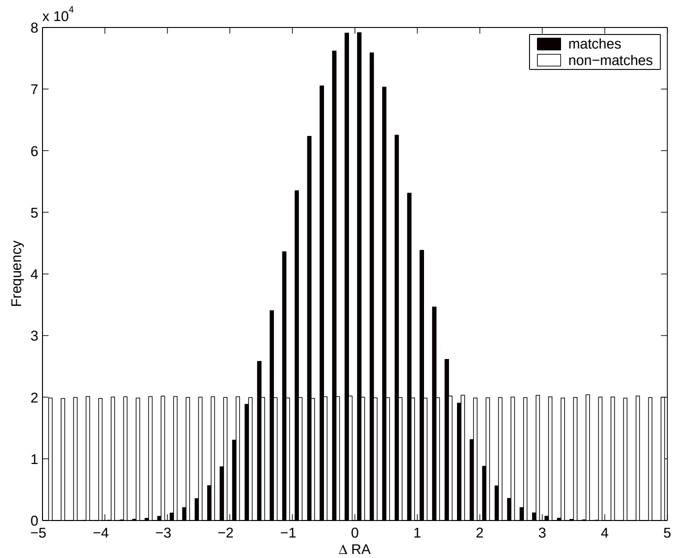


Figure 8. Our problem is to determine which objects match given position. From this histogram it is clearly impossible to establish this with certainty, and that the optimal procedure given any pair of links is to pick the one with the smallest proximity.

unusually high proximity objects that were discarded exactly for that reason: they are not typical matches. This has interesting implications for using catalogue matching to search for rare objects which is seen as one of the avenues for new science in the VO (Djorgovski et al. 2001) and in which the ClassX work is starting to make some progress (Suchkov & Hanisch 2004).

This example just discussed exaggerates the biasing effects, at least with respect to our HIPASS-SuperCOSMOS dataset. The error rate of 16 per cent in comparison to the problem used in this study is unrealistically high. The error rate in this simulation is determined by the variance of the Gaussian and the threshold where objects are not included (beyond ± 5). If we alter this by reducing the standard deviation to 0.1 then the error rate becomes 1.6 per cent and we find that the recovered distribution (Fig 9 (c)) is near Gaussian and we can more or less disregard false positives (Fig 9 (f)) and false negatives (Fig 9 (i)), note the change of scale on the x-axes in these figures.

A reasonable amount of knowledge of the underlying distributions is required in order to match objects together. In the approach taken here we actually choose to act as if our density estimates are completely true (of course they are not). In this situation where we already have a great deal of information what do we gain by using our newly matched data? Why not simply use the linked subset for inference and skip the complexity of this matching step? Precisely how much information do we lose by ignoring some of our data? A possible way to consider this problem is to use information theoretic concepts such as *information gain*. In this situation the problem of scientific inference is seen as equivalent to constructing a communication channel which can transmit the scientific 'truth' using the smallest number of bits. Constructing a communication channel using probabilities from the matched subset is likely to work reasonably well. However a more efficient communication channel would also employ the hard-won information from the newly matched data. The difference in the efficiency of the two communication channels has intuitive appeal as the information gain. In a Bayesian approach this additional information is calculated as the Kullback-Leibler divergence between the prior and the posterior. This is a common heuristic for measuring the expected information gain see (Lindley 1956; Bernardo 1979). In our application if the expected information gain is low it seems reasonable that the considerable effort to gain information from catalogue matching may not be worthwhile. In future work we plan to investigate this further.

6 SUMMARY

In this paper we presented a way to view the statistical problem of catalogue matching or linkage in terms of a combination of either generative pdf estimators or discriminative pattern classifiers. The discriminative method however fails to include a very reasonable independence assumption. We applied each approach to the problem of matching HIPASS-SuperCOSMOS and showed two approaches to using all available information. While fitting these models we were unable to provide an absolute criterion for an optimal probabilistic estimate, however we provided a number of heuristics to test if the probabilities are satisfactory.

Probabilities are useful as they allow completeness efficiency trade-offs to be controlled, they also make it possible to search for rare objects. A list of 30 dark galaxy candidates were provided using probabilities in order to produce a ranked list.

The study produced here is in some sense easier than many matching tasks that may be performed in that a training set

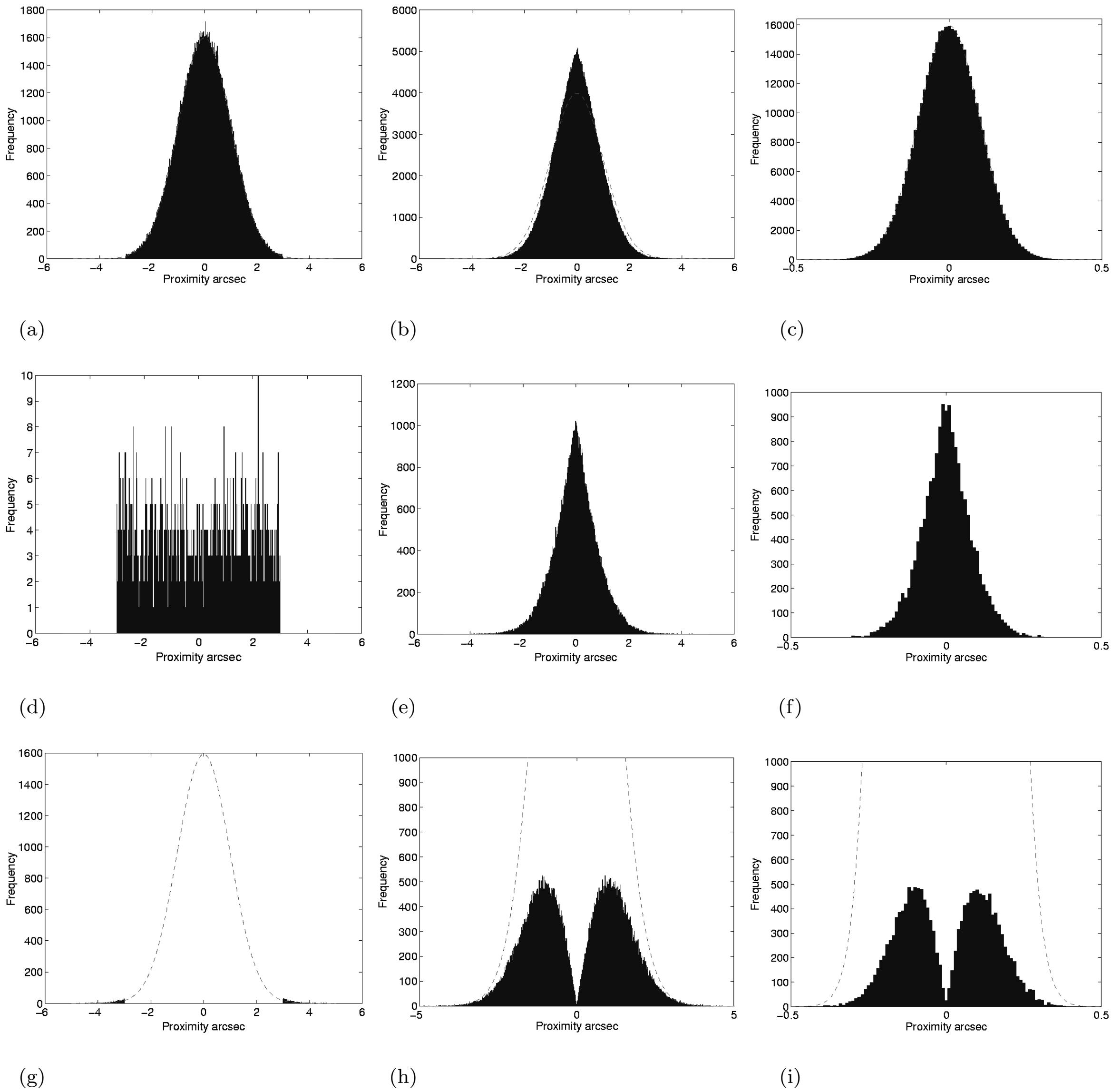


Figure 9. The distributions recovered from our simple matching simulation. The thresholding output is shown in (a) the false positives in (d) and the false negatives in (g) we note that the output distribution is a good recovery of the Gaussian distribution (dashed line) the cost of this is that 30 per cent of data was discarded which is a significant waste of information. The output by selecting the most likely match is shown in (b) this is noticeably different from Gaussian - it is a tighter distribution, the false positives are shown in (e) and the false negatives in (h). These simulations dwell on a pessimistic case, if we attempt an easier problem where the standard deviation of the Gaussian is reduced from 1 arcsec to 0.1 arcsec the error for the most likely match procedure drops to 1.6 per cent. The output is then satisfactorily close to Gaussian (c) and the false positives (f) and false negatives (i) are (perhaps) negligible.

of matched examples was already available [5]. The availability of a training set allows the density estimation to be performed in high dimensional space using semi-parametric models such as Gaussian Mixture Models. This allows a direct application of the Sutherland & Saunders (1992) formalism which appears to give reasonable results. From a machine learning perspective this would be termed a generative model.

On-going debate in machine learning and statistics communities continues on the relative merits of generative and discriminative methods for classification. The discriminative method is often preferred in the literature, however for our problem it is non-trivial to obtain a probability conditioned on all available parameters. It also causes an independence assumption to be ignored that we know applies a priori. Despite this, the classification results of a probabilistic (discriminative)

[5] In order to construct a model without training data, Sutherland & Saunders (1992) recommended a procedure that involved histogram subtraction of two distributions although this has been found to give unsatisfactorily noisy estimates (Mann et al. 1997). For a more sophisticated way of estimating these densities see Storkey et al. (2005)

SVM are very competitive with the generative model. In terms of classification the (discriminative) SVM had a slight edge over the generative Gaussian Mixture Model.

It turns out that, more generally, the question of 'is this probability a good probability' is not well posed. We offer three heuristics checks, classification rates, calibration diagrams and Brier scores for evaluating the quality of the probabilities, but none could be considered definitive. Both the SVM and Gaussian Mixture Model performed well against these intuitive measures; the question is not sufficiently well posed to make any statement about one being better than the other.

The utility of probabilities was demonstrated in that it was possible to produce a matched catalogue with some control over completeness and efficiency. Moreover probabilities were useful in obtaining a list of candidate dark galaxies for which follow up observation with the Australia Telescope Compact Array may be informative.

This paper generalises the framework of Sutherland & Saunders (1992) to deal with sparse parameters as well as dense parameters and considers the problem using high dimensional pdfs. The problem of estimating the distributions of interest is open, however we have shown two alternative methods both can provide good results. We are considering another approach using Bayesian inference approximated using Markov Chain Monte Carlo algorithms for a future paper.

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APPENDIX A - THE SUTHERLAND AND SAUNDERS RESULT

Assuming that each a links to at most one b then we can say:

$$P(\alpha_x, \beta_{x,1}, \cdots, \beta_{x,N_x}|z_{x,k} = 1) = P(\alpha_x, \beta_{x,k}|z_{x,k} = 1) \prod_{i=1..N_x, i\neq k} P(\alpha_x, \beta_{x,i}|z_{x,j} = 0). \tag{10}$$

Similarly if a links to *no* b then we can say:

$$P(\alpha_x, \beta_{x,1}, \cdots, \beta_{N_x}|z_{x,1} = 0, \cdots, z_{x,N_x} = 0) = \prod_{i=1..N_x} P(\alpha_x, \beta_{x,i}|z_{x,i} = 0). \tag{11}$$

The quantity of interest is

$$P(z_{x,y} = 1|\alpha_x, \beta_{x,1} \cdots, \beta_{x,N_x}) = \frac{P(\alpha_x, \beta_{x,1}, \cdots, \beta_{x,N_x}|z_{x,y}=1)P(z_{x,y}=1)}{\sum_{j=1..N_x} P(\alpha_x, \beta_{x,1}, \cdots, \beta_{x,N_x}|z_{x,j}=1)P(z_{x,j}=1) + P(\alpha_x, \beta_{x,1}, \cdots, \beta_{x,N_x}|z_{x,1}=0, \cdots, z_{x,N_x}=0)P(z_{x,1}=0, \cdots, z_{x,N_x}=0)} \tag{12}$$

Assuming the priors belief is the same ($\frac{1}{N+\kappa}$) for all candidates. It follows that the prior belief for no match is $\frac{\kappa}{N+\kappa}$.

$$P(z_{x,y} = 1|\alpha_x, \beta_{x,1}, \cdots, \beta_{x,N_x}) = \frac{P(\alpha_x, \beta_{x,1}, \cdots, \beta_N|z_{x,y} = 1)\frac{1}{N_x+\kappa}}{\sum_{j=1..N_x} P(\alpha_x, \beta_{x,1}, \cdots, \beta_{x,N_x}|z_{x,j} = 1)\frac{1}{N_x+\kappa} + P(\alpha_x, \beta_{x,1}, \cdots, \beta_{x,N_x}|z_{x,1} = 0, \cdots, z_{x,N_x} = 0)\frac{\kappa}{N_x+\kappa}} \tag{13}$$

This simplifies to

$$P(z_{x,y} = 1|\alpha_x, \beta_{x,1}, \cdots, \beta_{x,N_x}) = \frac{P(\alpha_x, \beta_{x,1}, \cdots, \beta_{N_x}|z_{x,y} = 1)}{\sum_{j=1..N_x} P(\alpha_x, \beta_{x,1}, \cdots, \beta_{x,N_x}|z_{x,j} = 1) + P(\alpha_x, \beta_{x,1}, \cdots, \beta_{x,N_x}|z_{x,1} = 0, \cdots, z_{x,N_x} = 0)\kappa} \tag{14}$$

Substituting 10 and 11 into 13 we get

$$P(z_{x,y} = 1|\alpha_x, \beta_{x,1}, \cdots, \beta_{x,N_x}) = \frac{P(\alpha_x, \beta_{x,k}|z_{x,k}=1)\prod_{i=1..N_x, i\neq k} P(\alpha_x, \beta_{x,i}|z_{x,j}=0)}{\sum_{j=1..N_x} P(\alpha_x, \beta_{x,j}|z_{x,j}=1)\prod_{i=1..N_x, i\neq j} P(\alpha_x, \beta_{x,i}|z_{x,i}=0) + \prod_{i=1..N_x} P(\alpha_x, \beta_{x,j}|z_{x,j}=0)\kappa} \tag{15}$$

Divide top and bottom by $\prod_{i=1..N_x} P(\alpha, \beta_i|z_{x,i} = 0)$

$$P(z_{x,y} = 1|\alpha_x, \beta_{x,1}, \cdots, \beta_{x,N_x}) = \frac{\frac{P(\alpha_x, \beta_{x,y}|z_{x,y}=1)}{P(\alpha_x, \beta_{x,y}|z_{x,y}=0)}}{\sum_{j=1..N_x} \frac{P(\alpha_x, \beta_{x,j}|z_{x,j}=1)}{P(\alpha_x, \beta_{x,j}|z_{x,j}=0)} + \kappa} \tag{16}$$

The above is the likelihood ratio result from Sutherland & Saunders (1992). As argued in the body of this document another useful form for this equation is:

$$P(z_{x,y} = 1|\alpha_x, \beta_{x,1}, \cdots, \beta_{x,N_x}) = \frac{\frac{1}{P(z_{x,y}=1|\alpha_x, \beta_{x,y})^{-1}-1}}{\sum_{j=1..N_x} \frac{1}{P(z_{i,j}=1|\alpha_x, \beta_{x,j})^{-1}-1} + \frac{P(z_{i,j}=1)}{P(z_{i,j}=0)}\kappa} \tag{17}$$

the advantage being that a neural network or Platt calibrated SVM return a probability of the form $P(z_{x,y} = 1|\alpha_x, \beta_{x,y})$.